NTA Annual Conference on Taxation

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How Much Is Enough?

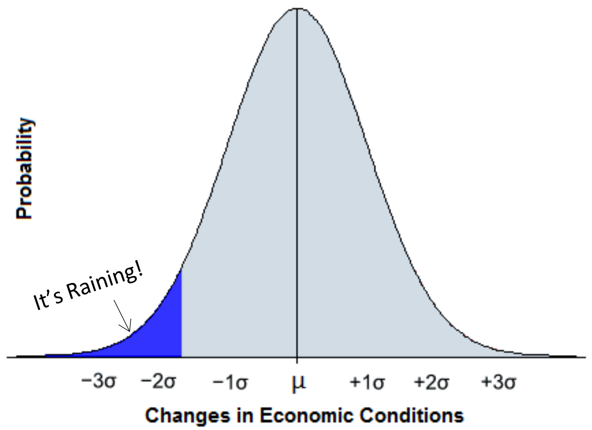
State Rainy Day Funds & Tax System Volatility

How Much Is Enough?

State Rainy Day Reserves & Tax System Volatility

Most state governments (including Minnesota) use rainy day reserves to cushion against fiscal stress caused by changing economic conditions.

Yet public finance literature offers little empirical guidance on the amount of savings a state needs (Joyce 2001).



Our research attempts to provide a statistical method for estimating the appropriate size rainy day fund by quantifying the volatility of a state's tax base – over time.

Key Motivations

- **Predictable qualities of a tax base depend on both structural trend growth and short-term (or cyclical) disturbances** (Hodrick and Prescott 1997)
- **Revenue volatility (standard deviation) changes over time** (Engle and Bollerslev 1986; Garrett 2009)
- **Interaction (correlation) between tax components changes over time** (Engle 2002)
- **State revenue characteristics treated like a portfolio of assets** (White 1983; Markowitz 1952)

Outline

1. **Process**
   - Empirical Considerations
     - Measuring Techniques
     - Time Period
   - Minnesota's Tax System
     - Description
     - Data Sources
2. **Method & Results**
   - Step 1: Detach Cyclical Deviation from Structural Trend Growth Rate
     Method: Hodrick-Prescott filter
   - Step 2: Construct Time-Varying Measures of Cyclical Volatility.
     Method: Integrated Generalized Autoregressive Conditional Heteroskedasticity (IGARCH) model
   - Step 3: Measure Time-Varying Covariation between Components
     Method: Integrated form of Dynamic Conditional Correlation (DCC) model
   - Step 4: Quantify System-Wide Volatility Over Time
     Method: Portfolio Choice model
3. **Estimating Appropriate Size Rainy Day Fund**

Process:

Empirical Considerations

Examine tax base (not revenues)

- Difficult to obtain a series of state revenue data uninfluenced by changes to tax law over time
- Data on tax bases as opposed to actual collections provides a reasonable substitute

Examine national tax base and income data

- Availability of detailed state-level economic data is limited
- National data serve as an appropriate proxy for state activity

Values are in nominal dollars

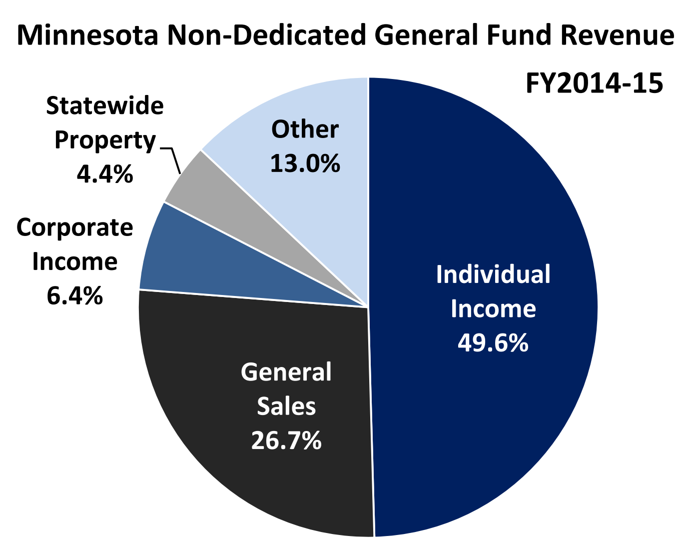
- Government budget authority is concerned with the growth and volatility of current dollar rates
- Converted to growth rates using log-differences (stationary)

Measure between 1965 and 2012

Process:

Minnesota's Tax System

Major Sources of Revenue:



1. **Individual Income Tax**

   Analyze 6 different personal taxable income types (before deductions) from IRS's *Statistics of Income (SOI)*
   - Salaries and wages
   - Taxable interest
   - Ordinary dividends
   - Net capital gains
   - Business-related income
   - All other taxable income

2. **General Sales Tax**

   Analyze 5 purchase categories from the *National Income and Product Accounts (NIPAs)* of the Bureau of Economic Analysis (BEA)
   - Consumer spending on non-auto durable goods
   - Non-durable goods subject to tax
   - Investment and government consumption
   - Household operation services
   - Other services subject to tax

3. **Corporate Income Tax** - **Pre-tax domestic corporate profits from the BEA *NIPAs***

4. **Statewide Property Tax** - **Implicit price deflator for state and local government consumption expenditures and gross investment from the BEA *NIPAs***

5. **Other Taxes and Fees & Tax Portfolio Shares** – **Derived from U.S. Census Bureau's State *Government Finance statistics* (adjusted for major changes in tax rates and bases).**

Empirical Strategy & Results

Step 1: Detach Cyclical Deviation from Structural Trend Growth Rate

Method: Hodrick-Prescott Filter (HPF)

HPF begins with an assumption that the growth rate of the *i*th tax $TAX_{i,t}$ can be decomposed into the sum of a long-term trend, $g_{i,t}$, and excess cyclical, $cyc_{i,t}$, growth rates at time *t*:

(1) $TAX_{i,t} = g_{i,t} + cyc_{i,t}$, for $t = 1,\ldots,T$ and $i = 1,\ldots,N$,

where T is the number of observations and N is the number of taxes.

A smooth non-linear representation of the trend growth rate component is obtained by minimizing the sum of the squared deviations of $TAX_{i,t}$ from $g_{i,t}$, subject to a penalty that restricts the sum of squared second differences of $g_{i,t}$:

(2) $$Minimize_{\{g_{i,t}\}_{t=1}^{T}} \left\{\sum_{t=1}^{T}\left(TAX_{i,t} - g_{i,t}\right)^2 + \lambda_i^{HP}\sum_{t=2}^{T-1}\left[\left(g_{i,t+1} - g_{i,t}\right) - \left(g_{i,t} - g_{i,t-1}\right)\right]^2\right\},$$

where $\lambda_i^{HP} > 0$ is a constant penalty parameter that controls the smoothness of the trend growth rate variations.

We use somewhat less conventional smoothing value $\lambda_i^{HP} = 400$ filter to capture greater persistence in data cycles.

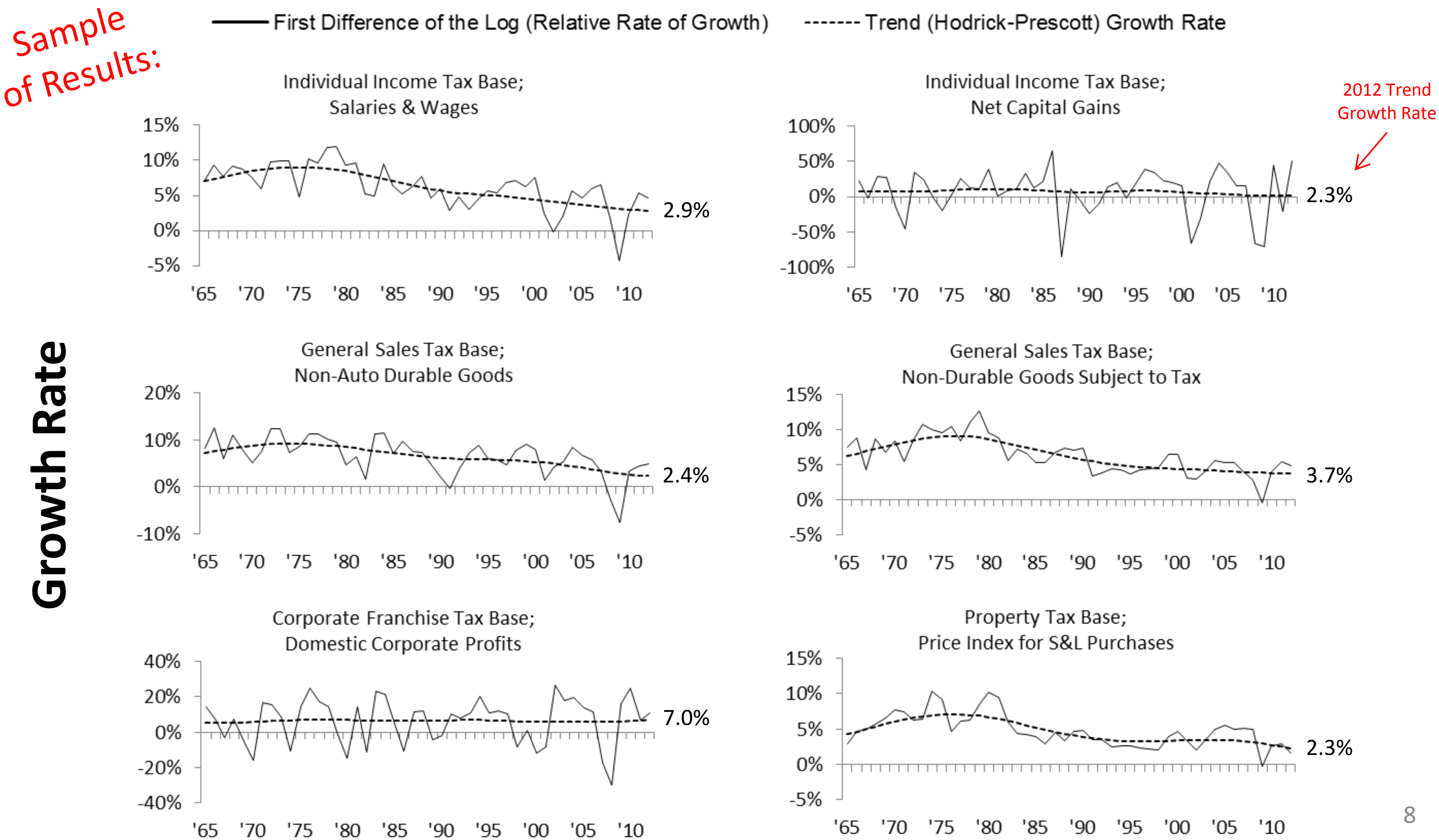
(Following Hodrick and Prescott 1997; Baxter and King 1999)

Empirical Strategy & Results

Step 1: Detach Cyclical Deviation from Structural Trend Growth Rate

Method: Hodrick-Prescott Filter

Figure 1: Growth Characteristics of Components in Minnesota's Tax Base



Empirical Strategy & Results

Step 2: Construct a Time-Varying Measure of Cyclical Volatility

Method: Integrated Generalized Autoregressive Conditional Heteroskedasticity (IGARCH) model

(Following Engle & Bollerslev 1986)

To illustrate how volatility transforms over time, excess cyclical errors from the HPF decomposition are used to construct a univariate IGARCH(1,1) conditional variance,$\boldsymbol{\sigma^2_{i,t}}$, for each detailed component of Minnesota's tax base.

For simplicity, the mean EQ(1) is re-specified so that short run cyclical deviations, $\boldsymbol{cyc_{i,t} = TAX_{i,t} - \ g_{i,t}}$, are modeled as the dependent variable containing only an intercept:

(3) $$cyc_{i,t} = \ \varphi_i^{\tau} + \ \varepsilon_{i,t}$$

where φ_i^{τ} is a constant parameter, $\varepsilon_{i,t}$ follows a time-varying heteroskedastic error pattern ~ i.i.d. N(0,$\sigma^2_{i,t}$), and $\varepsilon_{i,t} \ = \ \sigma_{i,t} Z_{i,t}$, where $Z_{i,t}$ ~ i.i.d. N(0,1)

The IGARCH(1,1) conditional variance equation is defined as one-period lag of forecast variance, $\boldsymbol{\sigma^2_{i,t-1}}$, and the squared residual, $\boldsymbol{\varepsilon^2_{i,t-1}}$, from the mean equation EQ(3):

(4) $$\sigma^2_{i,t} \ = \lambda_i^{IGARCH} \sigma^2_{i,t-1} + (1 - \lambda_i^{IGARCH}) \varepsilon^2_{i,t-1}.$$

where $\lambda_i^{IGARCH} > 0$ is a constant IGARCH parameter for each unique *i*th tax.

Empirical Strategy & Results

Step 2: Construct a Time-Varying Measure of Cyclical Volatility

Method: Integrated Generalized Autoregressive Conditional Heteroskedasticity (IGARCH) model

Table 1: Conditional Variance IGARCH(1,1) Model Results of Minnesota's Tax Base

Dependent Variable	Model	ARMA(s)	β	α	Distribution	t-DOF	Skewness	Kurtosis	JB	Q(6)	Q(12)	Q(18)	$Q^2(6)$	$Q^2(12)$	$Q^2(18)$	ARCH-LM(5)
Corporate Franchise Tax Base																
Domesitc Corporate Profits (CORP)	IGARCH(1,1)	MA(1)	0.781	0.219	Normal	-	-0.442	-0.602	(0.303)	(0.201)	(0.386)	(0.283)	(0.244)	(0.683)	(0.877)	(0.383)
			(0.000)	(0.045)												
General Sales Tax Base (SALES)																
Non-Auto Durable Goods (CDXMV)	IGARCH(1,1)	MA(1)	0.932	0.068	Normal	-	-0.742	0.521	(0.045)	(0.167)	(0.561)	(0.600)	(0.269)	(0.576)	(0.676)	(0.286)
			(0.000)	(0.049)												
Non-Durable Goods Subject to Tax (CNO)	IGARCH(1,1)	AR(1)	0.889	0.111	Normal	-	-0.708	1.555	(0.004)	(0.250)	(0.474)	(0.856)	(0.899)	(0.859)	(0.913)	(0.909)
			(0.000)	(0.016)												
Household Operation Services (CSVH)	IGARCH(1,1)	AR(1)	0.879	0.121	Student's t	5.290	0.945	1.372	(0.001)	(0.382)	(0.123)	(0.265)	(0.476)	(0.895)	(0.963)	(0.204)
			(0.000)	(0.048)		(0.015)										
Other Services Subject to Tax (CSVO)	IGARCH(1,1)	-	0.885	0.115	Student's t	4.017	-0.448	2.026	(0.002)	(0.482)	(0.805)	(0.933)	(0.554)	(0.944)	(0.918)	(0.668)
			(0.000)	(0.050)		(0.005)										
Investment & Government Purchases (INV)	IGARCH(1,1)	MA(1)	0.714	0.286	Normal	-	-1.023	0.786	(0.005)	(0.332)	(0.514)	(0.712)	(0.856)	(0.997)	(0.997)	(0.910)
			(0.000)	(0.030)												
Individual Income Tax Base (IND)																
Salaries & Wages (WSD)	IGARCH(1,1)	-	0.805	0.195	Normal	-	-0.676	0.362	(0.125)	(0.198)	(0.430)	(0.577)	(0.464)	(0.160)	(0.314)	(0.765)
			(0.000)	(0.031)												
Taxable Interest (INT)	IGARCH(1,1)	MA(1)	0.863	0.137	Normal	-	0.330	0.316	(0.517)	(0.345)	(0.657)	(0.160)	(0.418)	(0.542)	(0.678)	(0.611)
			(0.000)	(0.029)												
Ordinary Dividends (DIV)	IGARCH(1,1)	MA(1)	0.725	0.275	Normal	-	-0.321	-0.096	(0.596)	(0.088)	(0.118)	(0.162)	(0.510)	(0.667)	(0.512)	(0.430)
			(0.000)	(0.029)												
Net Capital Gains (CG)	IGARCH(1,1)	-	0.837	0.163	Student's t	4.156	-0.794	1.561	(0.006)	(0.462)	(0.551)	(0.235)	(0.235)	(0.492)	(0.230)	(0.290)
			(0.000)	(0.004)		(0.016)										
Business-Related Income (BUS)	IGARCH(1,1)	AR(1)	0.736	0.264	Normal	-	0.266	-0.340	(0.639)	(0.233)	(0.380)	(0.448)	(0.659)	(0.542)	(0.509)	(0.839)
			(0.000)	(0.001)												
All Other Taxable Income (O)	IGARCH(1,1)	-	0.743	0.257	Normal	-	-0.036	-0.268	(0.919)	(0.507)	(0.702)	(0.905)	(0.417)	(0.290)	(0.345)	(0.455)
			(0.000)	(0.000)												
Property Tax Base																
Price Index for S&L Purchases (PROP)	IGARCH(1,1)	-	0.842	0.158	Normal	-	0.659	0.433	(0.119)	(0.112)	(0.341)	(0.381)	(0.770)	(0.785)	(0.830)	(0.947)
			(0.000)	(0.011)												
Other Tax Base																
Other General Fund Tax Revenue (OREV)	IGARCH(1,1)	-	0.791	0.209	Student's t	6.485	0.245	1.622	(0.050)	(0.242)	(0.290)	(0.194)	(0.899)	(0.930)	(0.986)	(0.892)
			(0.000)	(0.054)		(0.064)										
Tax-Exempt, Non-General Fund, and Other Series of Interest																
Food Products (CNF)	IGARCH(1,1)	-	0.817	0.183	Normal	-	-0.335	-0.688	(0.341)	(0.075)	(0.266)	(0.090)	(0.970)	(0.999)	(0.999)	(0.916)
			(0.000)	(0.005)												
Clothing and Shoes (CNCS)	IGARCH(1,1)	-	0.836	0.164	Normal	-	-0.455	-0.454	(0.263)	(0.240)	(0.382)	(0.712)	(0.868)	(0.913)	(0.968)	(0.854)
			(0.000)	(0.038)												
Pharmaceutical Products (CNDRUGS)	IGARCH(1,1)	MA(1)	0.879	0.121	Normal	-	0.613	0.094	(0.138)	(0.047)	(0.076)	(0.232)	(0.265)	(0.126)	(0.242)	(0.134)
			(0.000)	(0.013)												
Gasoline and Other Motor Fuels (CNGAS)	IGARCH(1,1)	-	0.818	0.182	Student's t	4.334	0.136	2.295	(0.003)	(0.163)	(0.396)	(0.429)	(0.887)	(0.976)	(0.993)	(0.810)
			(0.000)	(0.007)		(0.009)										
Motor Vehicles (CDMV)	IGARCH(1,1)	-	0.895	0.105	Student's t	3.402	-1.380	3.140	(0.000)	(0.558)	(0.814)	(0.668)	(0.993)	(0.999)	(0.998)	(0.987)
			(0.000)	(0.022)		(0.001)										
Personal Consumption Expenditures (PCE)	IGARCH(1,1)	AR(1)	0.677	0.323	Student's t	6.565	0.411	2.686	(0.000)	(0.377)	(0.384)	(0.538)	(0.648)	(0.958)	(0.994)	(0.852)
			(0.000)	(0.000)		(0.030)										

Notes: The table reports the maximum likelihood parameter estimates and descriptive statistics for the conditional variance equation of an IGARCH(1,1) specification for each detailed component of Minnesota's tax base. ARMA(s) denotes the autoregressive and moving average process applied to the residuals. β is the GARCH parameter in the conditional variance equation. α is the ARCH parameter in the conditional variance equation. The p-values for each coefficient of the variance equation are reported in parenthesis. Two distributions are considered for the error term: normal (Gaussian) distribution and the Student's t-distribution with the estimated t-DOF degrees of freedom. Skewness is a measure of positive or negative asymmetry of the distribution around zero (the mean). Positive skewness means the right tail of the distribution is longer while negative skewness implies the left tail of the distribution is longer. Kurtosis is the degree of "peakedness" or flatness relative to the normal distribution (equal to zero). Positive kurtosis means the distribution has a sharper peak and fatter tails than a normal distribution while negative kurtosis implies the distribution has a more rounded peak and thinner tails. JB is the reported p-value from a Jarque-Bera test with a null hypothesis that the model residuals are normally distributed. Because the JB test is known to be overly sensitive to outliers in small samples, Bollerslev-Wooldridge robust standard errors are computed when conditional normal is chosen as the error distribution. Q(k) and Q2(k) are the reported p-values for a Ljung-Box portmanteau test of residual autocorrelation and conditional heteroskedasticity (ARCH effects) up to order k. ARCH-LM is the reported p-value from Engle's (1982) Lagrange Multiplier (LM) test with a null hypothesis that there exists no ARCH effects in the residuals up to order 5.

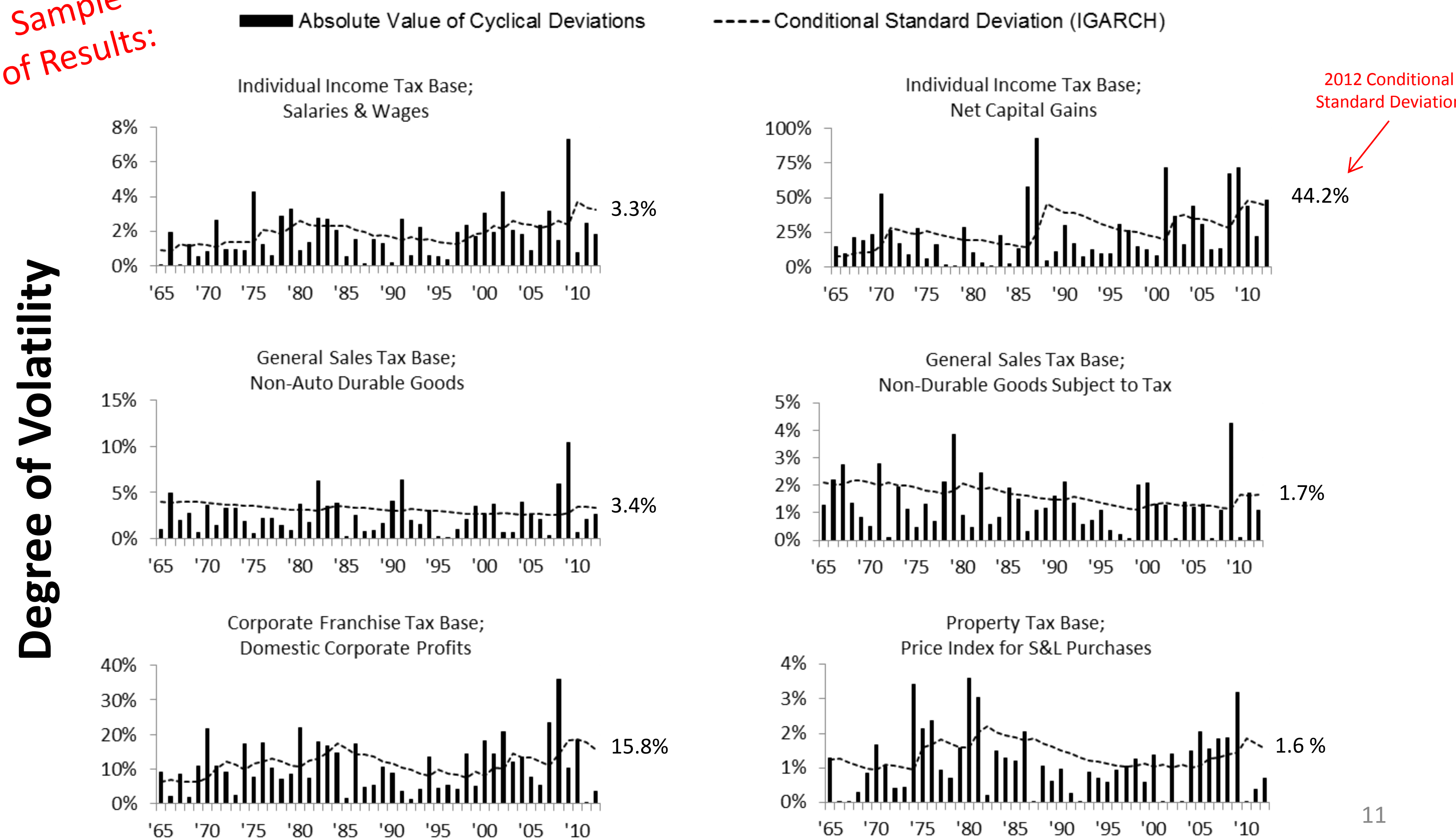
Empirical Strategy & Results

Step 2: Construct a Time-Varying Measure of Cyclical Volatility

Method: Integrated Generalized Autoregressive Conditional Heteroskedasticity (IGARCH) model

Sample of Results:

Figure 2: Volatility Characteristics of Components in Minnesota's Tax Base



Empirical Strategy & Results

Step 3: Measure Time-Varying Covariation between Components

Method: Integrated form of Dynamic Conditional Correlation (INT-DCC) model

The conditional correlation between two components *i* and *j* is also the conditional covariance of the standardized residuals from the IGARCH models defined in EQ(4).

Our analysis specifies these elements within a INT-DCC process:

(5) $$q_{ij,t} = \lambda^{INT-DCC}\left(q_{ij,t-1}\right) + \left(1-\lambda^{INT-DCC}\right)\left(Z_{i,t-1}Z_{j,t-1}\right),$$

where $Z_{i,t}$ is a standardized residual from the IGARCH models with $E\left[Z_{i,t}\right] = 0$ and $E\left[Z_{i,t}^2\right] = 1$ and $0 < \lambda^{INT-DCC} < 1$ is a constant scalar parameter for each unique combination of *i* and *j*, capturing the persistent effect of past standardized residuals and conditional covariance on current conditional covariance.

Finally, the conditional correlation estimator is given by:

(6) $$\rho_{ij,t} = \frac{q_{ij,t}}{q_{ii,t}q_{jj,t}},$$

where each term in the denominator has an expected value of 1.

Note: INT-DCC is programmed in Eviews using a log likelihood function

(Following Engle 2002)

Empirical Strategy & Results

Step 3: Measure Time-Varying Covariation between Components

Method: Integrated form of Dynamic Conditional Correlation (INT-DCC) model

Table 2: Integrated Dynamic Conditional Correlation (DCC-INT) Results of Minnesota's Tax Base

λ Parameter Estimates for Individual Income Tax Base

	WSD	INT	DIV	CG	BUS	O
WSD	-	0.101 (0.001)	0.027 (0.017)	0.145 (0.017)	0.156 (0.014)	0.081 (0.000)
INT		-	0.060 (0.012)	0.164 (0.000)	0.157 (0.006)	0.109 (0.000)
DIV			-	0.125 (0.007)	0.104 (0.012)	0.074 (0.018)
CG				-	0.151 (0.000)	0.168 (0.000)
BUS					-	0.030 (0.000)
O						-

λ Parameter Estimates for General Sales Tax Base

	CDXMV	CNO	CSVH	CSVO	INV
CDXMV	-	0.114 (0.035)	0.066 (0.019)	0.071 (0.007)	0.088 (0.008)
CNO		-	0.135 (0.006)	0.111 (0.035)	0.086 (0.025)
CSVH			-	0.137 (0.043)	0.117 (0.008)
CSVO				-	0.098 (0.000)
INV					-

λ Parameter Estimates for Total General Fund Tax Base

	IND	SALES	CORP	PROP	OTHER
IND	-	0.084 (0.008)	0.104 (0.000)	0.076 (0.004)	0.096 (0.012)
SALES		-	0.120 (0.001)	0.088 (0.002)	0.094 (0.013)
CORP			-	0.100 (0.014)	0.095 (0.002)
PROP				-	0.074 (0.020)
OTHER					-

Notes: Table reports a matrix of the log-likelihood parameter estimates for the conditional correlation estimator of an integrated Dynamic Conditional Correlation (DCC-INT) specification for joint components of Minnesota's tax base. The p-values for each coefficient of the DCC_INT equations are reported in parenthesis.

Empirical Strategy & Results

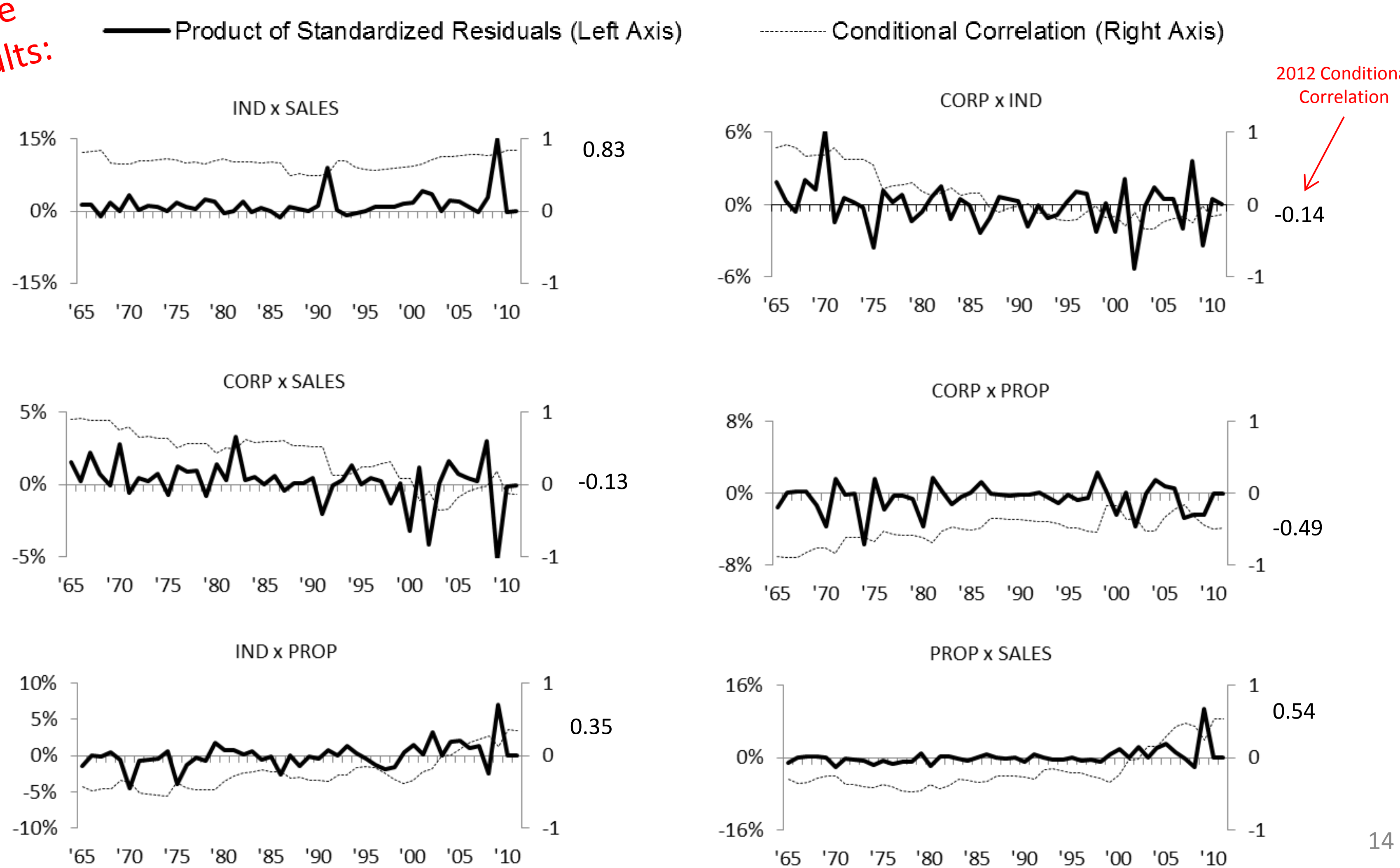
Step 3: Measure Time-Varying Covariation between Components

Method: Integrated form of Dynamic Conditional Correlation (INT-DCC) model

Figure 4: Degree of Correlation between Components in MInnesota's Tax Base

Sample of Results:

Degree of Covariation



Empirical Strategy & Results

Step 4: Quantify System-Wide Volatility Over Time

Method: Portfolio Choice model

The conditional volatility of the total tax system $\boldsymbol{\sigma_{P,t}}$ is expressed by modifying a Portfolio Choice model to include a dynamic element for time *t*:

(7) $$\sigma_{P,t} = \sqrt{\sum_{i=1}^{N} \sum_{j=1}^{N} w_{i,t}\, w_{j,t} \rho_{ij,t} \sigma_{i,t} \sigma_{j,t}}$$

where $w_{i,t}$ and $w_{j,t}$ are the weighted proportions of the ith and jth taxes at time t respectively, $\sigma_{i,t}$ and $\sigma_{j,t}$ are the conditional standard deviations from the univariate IGARCH(1,1) specification, and $\rho_{ij,t}$ is the degree of conditional correlation between the ith and jth taxes from the bivariate DCC-INT estimator.

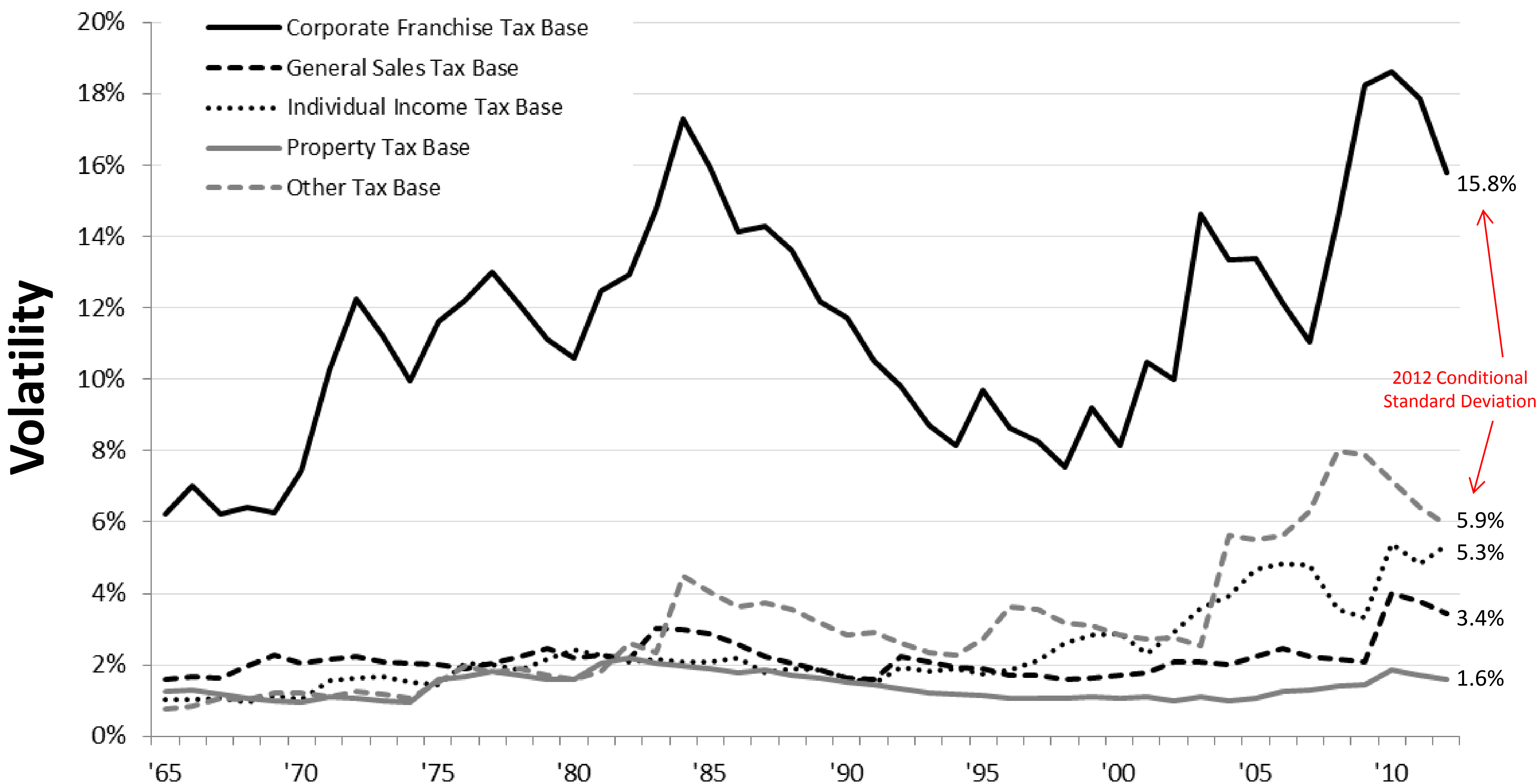
(Following White 1983)

Empirical Strategy & Results

Step 4: Quantify System-Wide Volatility Over Time

Method: Portfolio Choice model

Figure 6: Conditional Standard Deviation of Major Components of Minnesota's General Fund Tax Base

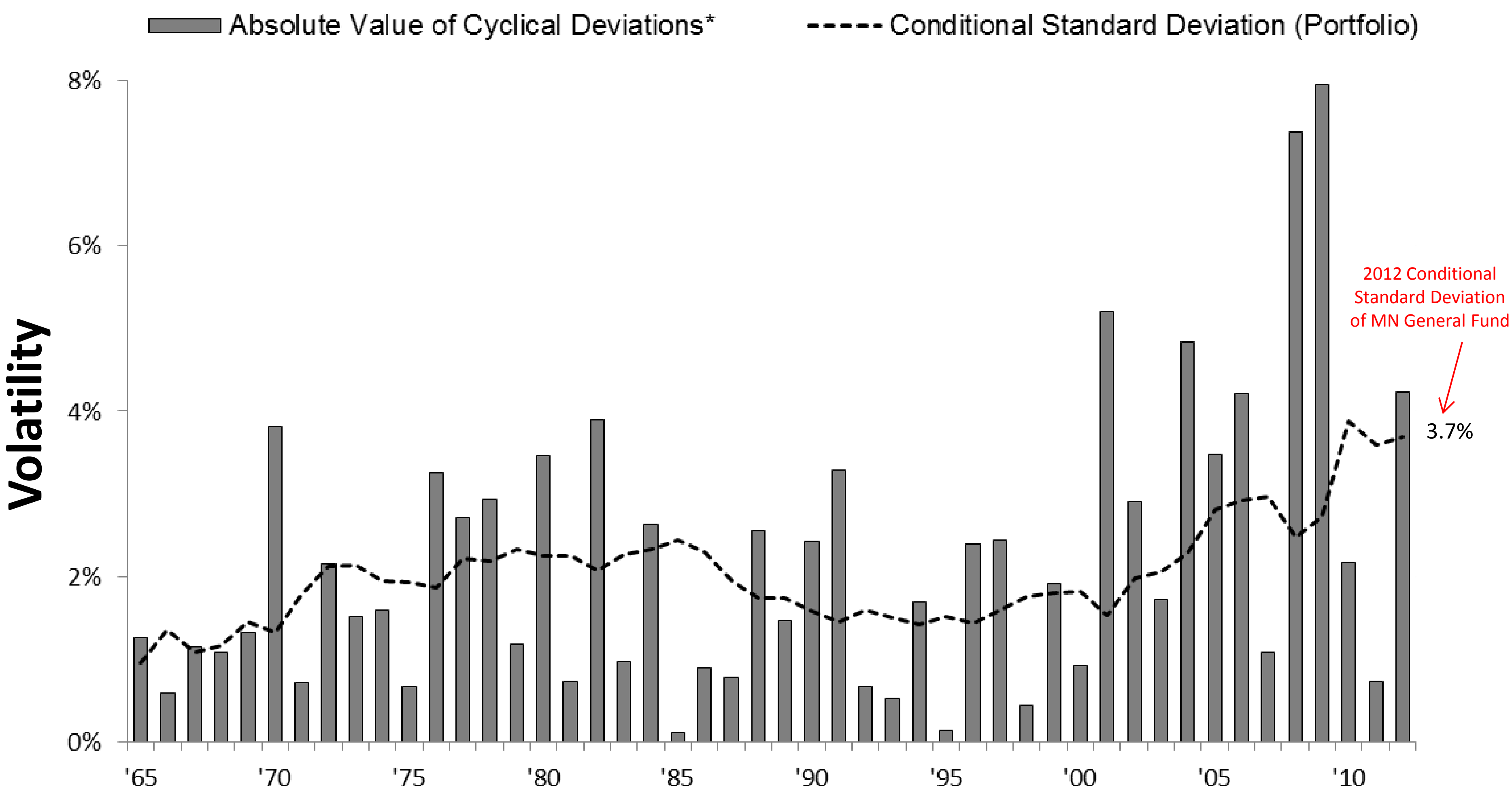


Empirical Strategy & Results

Step 4: Quantify System-Wide Volatility Over Time

Method: Portfolio Choice model

Figure 7: Conditional Standard Deviation of Minnesota's General Fund Tax Base Portfolio



* Calculated as the weighted sum of log differences less the weighted sum of trend growth rates.

Appropriate Size Rainy Day Fund?

Over time, an increasingly volatile tax base can have meaningful implications for state gov. finances & risk management strategies.

To protect against the prevailing level of risk, an appropriate size rainy day fund can be calculated:

1. **Convert estimated tax base volatility (σ = 3.7%) to revenue volatility (σ = 4.3%)**
   Reason: Progressivity in MN's individual income tax; elasticity w/ respect to tax base ≈ 1.3
2. **Choose failure rate: % of time deficit exceeds budget reserve (1 in 20, 5%)**
3. **Multiply existing revenue volatility measure by failure value. (4.3%*1.645 = 7.1%)**
   Note: Critical value for a 5% one tail z test from a normal distribution = 1.645
4. **MN budgets on a two-year basis: Use same procedure but for two years.**
   (7.1% * √2 = 10.1% of Annual Revenues)

Demonstrates that a state rainy day fund of <u>10.1% of annual revenues</u> will sufficiently protect against cyclical risk in all but 1 of every 20 two-year budget periods.

If $X \sim N(\mu_X, \sigma_X^2)$ and $Y \sim N(\mu_Y, \sigma_Y^2)$ are independent normal random variables, then $X + Y \sim N(\mu_X + \mu_Y, \sigma_X^2 + \sigma_Y^2)$

Summary

- **Examine detail components of state's tax base:**
  - **Individual Income - 6 sub-components**
  - **General Sales - 5 sub-components**
  - **Corporate Income, Property, and Other taxes and fees**
- **Measure system-wide volatility over time (1965-2012) by combining:**
  - **Hodrick-Prescott Filter** (Hodrick and Prescott 1997)
  - **Integrated Generalized Autoregressive Conditional Heteroskedasticity (IGARCH) model** (Engle and Bollerslev 1986; Garrett 2009)
  - **Integrated Dynamic Conditional Correlation (INT-DCC) model** (Engle 2002)
  - **Portfolio Choice model** (White 1983; Markowitz 1952)
- **Results:**
  - **Minnesota's tax base has grown more volatile since the late 1990s.**
  - **Attribute to growth in increasingly unstable components, such as corporate income and forms of individual investment income**
  - **State rainy day reserve of 10.1% of annual revenues will adequately protect against MN's cyclical risk.**

Thank You

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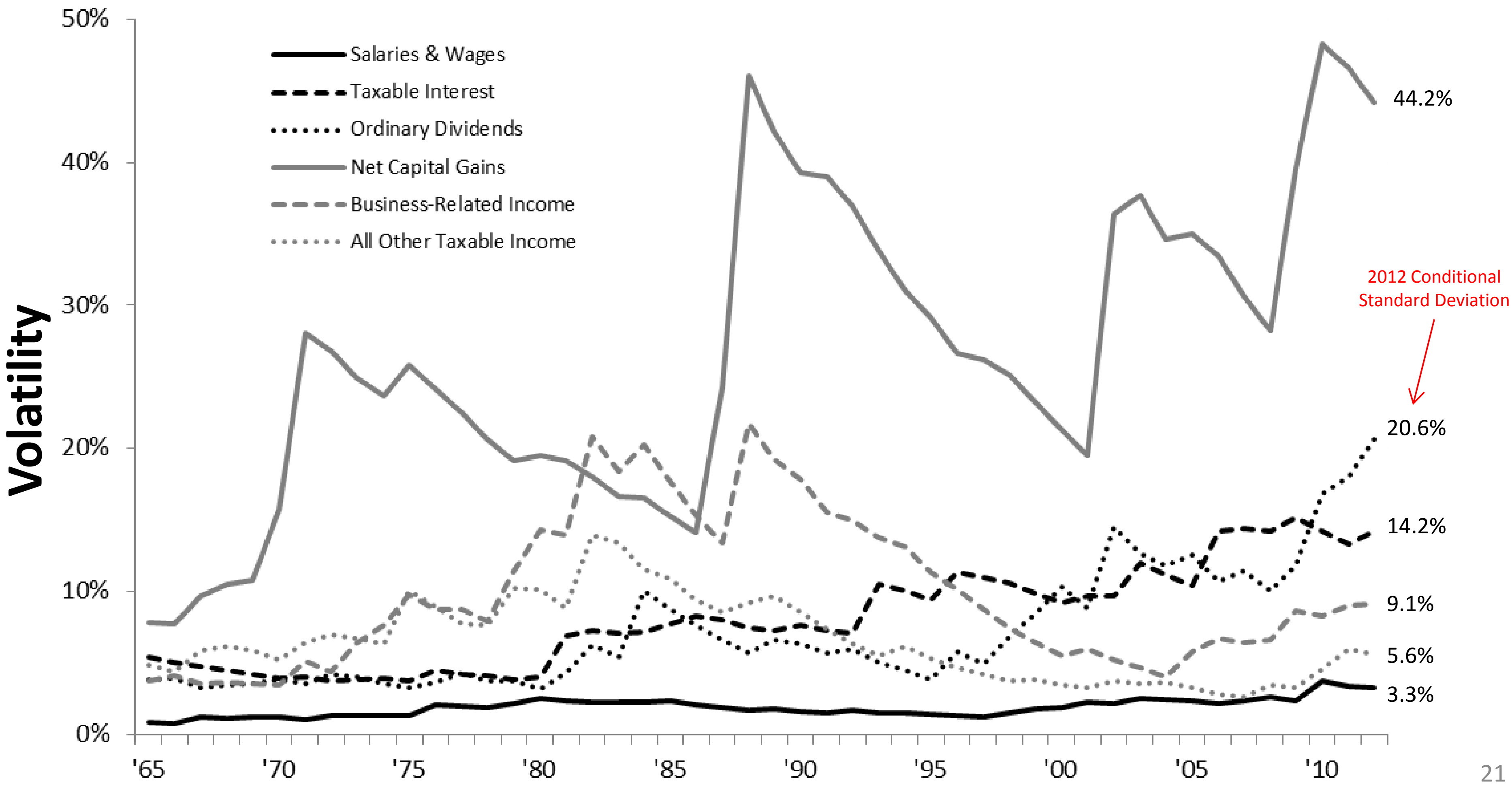
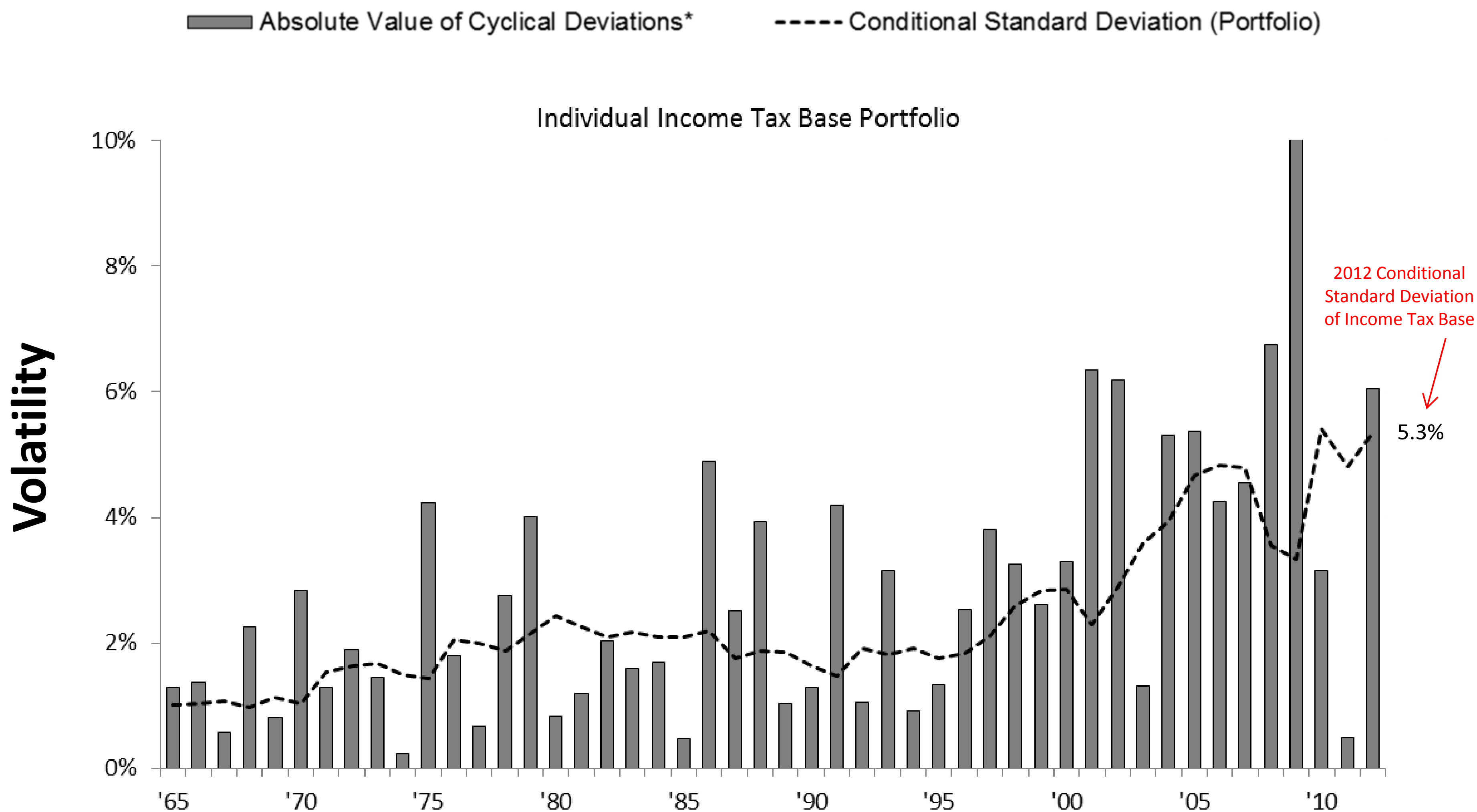
Supplemental Charts/Data

Figure 3a: Conditional Standard Deviation of Minnesota's Individual Income Tax Base Components



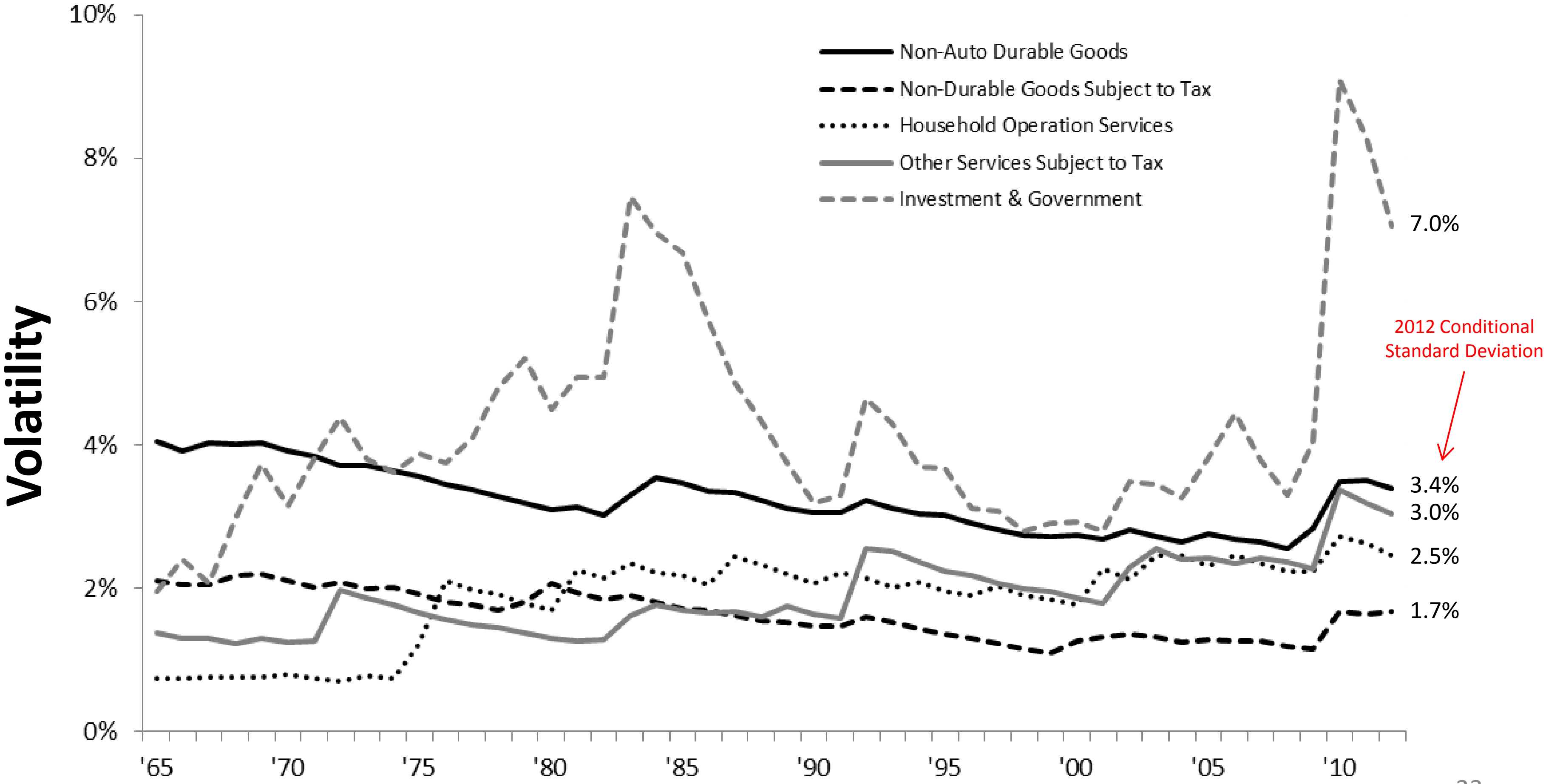
Supplemental Charts/Data

Figure 5: Time-Variant Volatility and the Portfolio Choice Model



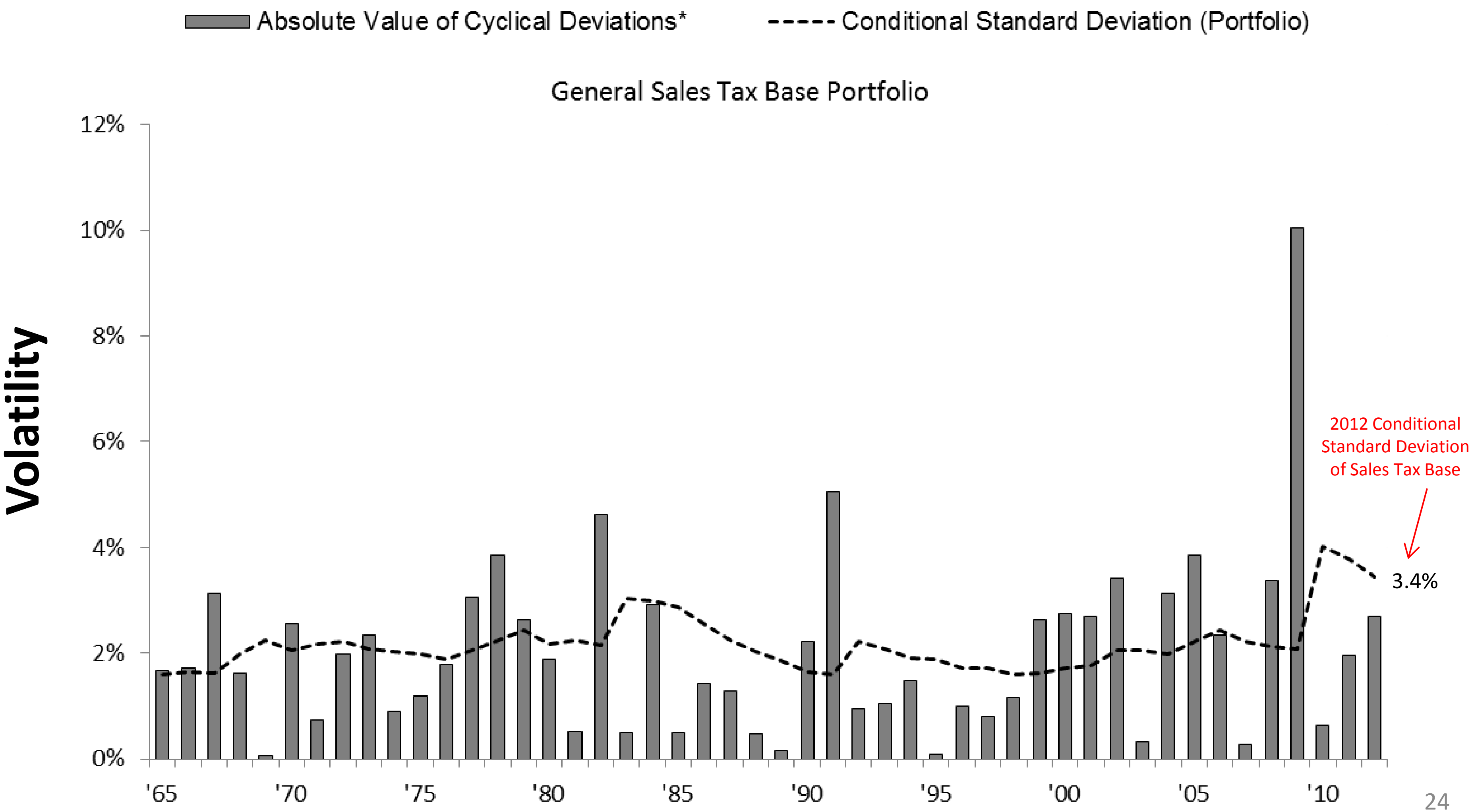
Supplemental Charts/Data

Figure 3b: Conditional Standard Deviation of Minnesota's General Sales Tax Base Components



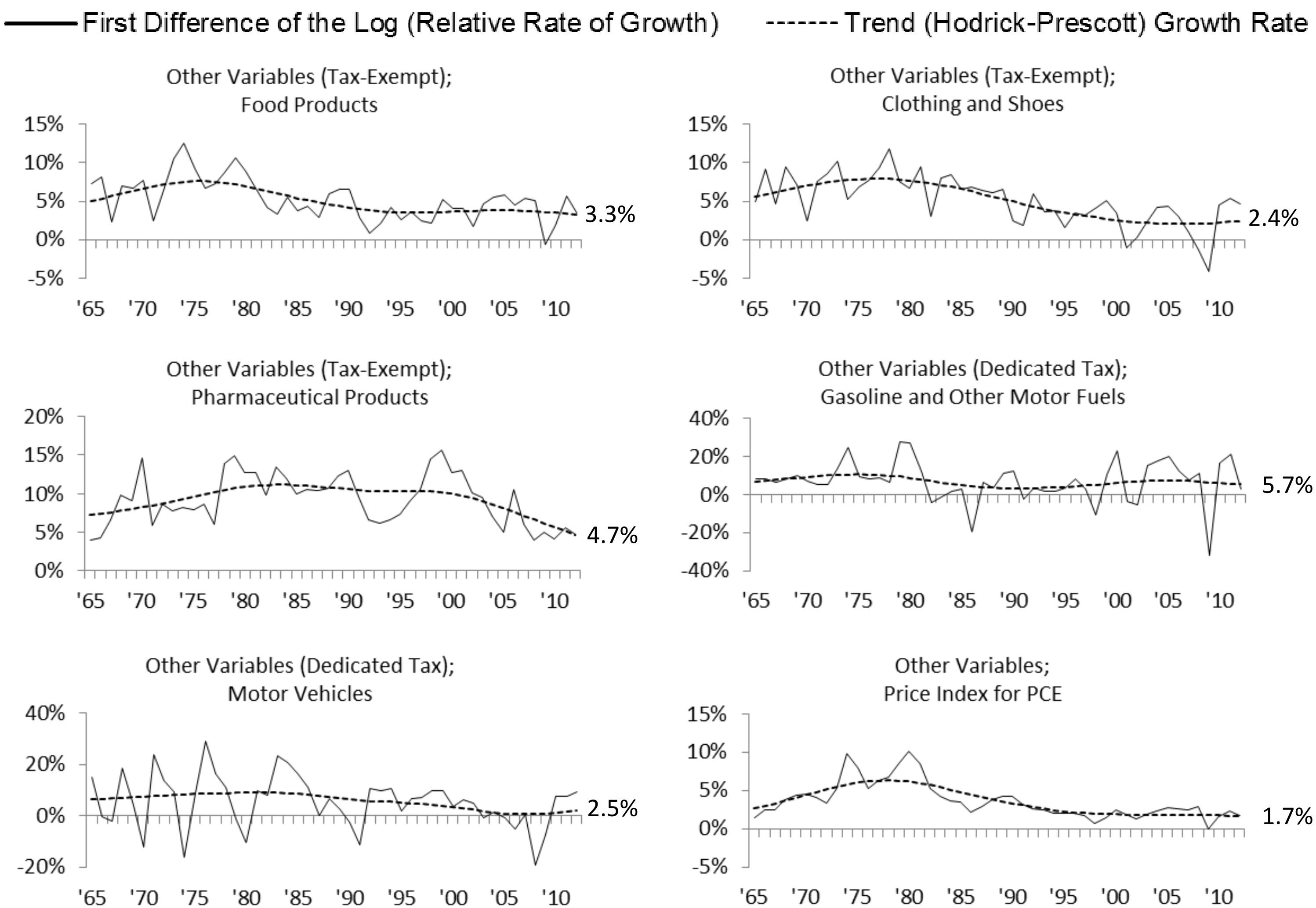
Supplemental Charts/Data

Figure 5: Time-Variant Volatility and the Portfolio Choice Model



Supplemental Charts/Data

Figure 8: Growth Characteristics of Other Variabes (Dedicated and Tax Exempt)



Supplemental Charts/Data

Figure 9: Volatility Characteristics of Other Variables (Dedicated and Tax Exempt)